Abstract

The interaction between labor unions and international competition has received a lot of attention. In the first decade of the 21st century, increasing number of companies in unionized oligopolistic industries has shifted production abroad. However, the issue of outsourcing in the context of a unionized duopoly has received little attention. In this paper, I model the option to outsource in the context of unionized labor markets in exporting industries. I show that if the inverse demand function is convex, then an increase in the foreign wage increases the utility of a wage neutral union. I find that if the domestic firm can credibly threaten to shift production abroad, then an increase in the outsourcing country’s wage may increase domestic profits if the union is labor-oriented.

Keywords: oligopoly, outsourcing, labor union

Unionized Oligopoly and Outsourcing

The interaction between labor unions and international competition has received a lot of attention. Brander & Spenser (1988) analyzed the effects of unionization in a Nash-Cournot duopoly market. They use the right to manage model, where the firm and union bargain over the wage first and then the firm sets its output level unilaterally. Mezzetti & Dinopoulos (1991) developed a Cournot duopoly model for a unionized domestic firm and a foreign firm. Mezzetti & Dinopoulos (1991) use the efficient bargaining model, where the firm and the union jointly bargain over wage and employment level. Mezzetti & Dinopoulos (1991) then abstracted a model for unionized labor markets in exporting industries.

In the first decade of the 21st century, an increasing number of companies in unionized oligopolistic industries, such as automobile and aircraft manufacturing, have shifted production abroad in an effort to cut labor costs. However, the issue of outsourcing within the context of a unionized duopoly has received very little attention. Mezzetti & Dinopoulos (1991) modeled an option to outsource and received limited results. They developed a model in which a unionized firm competes with a foreign firm for sales in the domestic market. Based on the Cournot model and terminology, a two-stage strategic game is used by the home country’s government, the union, and the two firms. In the first stage, the government announces a tariff imposed on the output of the foreign firm. In the second stage, the domestic firm and union jointly bargain over wage and employment level. Mezzetti & Dinopoulos found that a credible threat to shift production abroad increases domestic profits and lowers the negotiated wage. They then abstracted a model for unionized labor markets in exporting industries. I am unaware of any papers modeling an option to outsource in the context of unionized labor markets in exporting industries. Hence I present a model, which intends to fill the gap. I use the model to inquire into the effects of union’s bargaining.
power, foreign wage, foreign and domestic subsidy and wage in the outsourcing country.

The model uses a Stone-Geary representation which has been discussed extensively in the literature (Bandopadhyay & Bandyopadhyay, 2000; Dertouzos & Pencavel, 1981; Macurdy & Pencavel, 1986; Mezzetti & Dinopoulos, 1991; Pencavel, 1984).

I found that if the union is employment-oriented, then an increase in its bargaining power increases domestic output and the union’s utility level, but has an ambiguous effect on domestic profits. This result is similar to the results observed by Mezzetti & Dinopoulos (1991). I further found that if the union is employment-oriented, then an increase in its bargaining power may reduce domestic welfare, a result different to that of Mezzetti & Dinopoulos (1991) and Brander & Spencer (1988).¹

I find that if the domestic firm can credibly threaten to shift all of its production abroad, then an increase in the outsourcing country’s wage may decrease the union’s utility level and decrease the domestic welfare if the union is labor-oriented. In addition, this paper shows that if the inverse demand function is convex, then an increase in the foreign wage increases the utility level of a wage-neutral union.

**The Model**

In this model, two strictly exporting firms (domestic and foreign) are engaged in a duopolistic competition of a homogeneous good in a third nation’s market. The domestic firm is unionized and produces all of its output at the domestic plant; however, the domestic firm has an option to close the domestic plant and produce all of its output abroad in a fourth country. An example: Boeing in the US and Airbus in France may compete in Argentina’s market. Boeing is unionized but could shift all of its production to China if the union is overly demanding. The domestic firm’s local plant conducts an efficient Nash bargain to jointly determine the union and the domestic firm, the domestic firm can conduct an efficient Nash bargain to jointly determine the domestic wage, output, and labor combinations of the domestic and foreign firm.

Let us assume that the inverse demand function for the export good is:

\[ P = P(Q) = P(L + L^*) \]  

where \( P \) is the price, \( Q \) is the total quantity, \( L \) is the domestic output, and \( L^* \) is the foreign output.

The first order conditions with respect to \( w \) and \( L \) are:

\[ \alpha \pi U_w + (1-\alpha) \ U \pi_w = 0, \]  

\[ \alpha \pi U_L + (1-\alpha) \ U \pi_L = 0. \]  

It follows directly from the first order conditions that

\[ U = (w - b)^\beta L, \]  

where \( U \) is the utility of the domestic union, \( w \) and \( L \) are the wage rate and the employment level negotiated between the domestic union and the domestic firm, \( b \) is the alternative (competitive) wage and \( \beta \) is the degree of wage orientation of the domestic union. The union is wage-oriented if \( \beta > 1 \), wage-neutral if \( \beta = 1 \) and employment-oriented if \( \beta < 1 \). Pencavel (1984) has shown that there is empirical evidence that unions differ according to the degree of their wage orientation.

The subsidy inclusive domestic firm’s profit is:

\[ \Pi = \frac{P(L + L^*) - w + s}{L}, \]  

where \( w \) is the negotiated domestic wage and \( s \) is the domestic firm’s subsidy level.

The subsidy inclusive foreign firm’s profit is:

\[ \Pi^* = \frac{P(L + L^*) - w^* + s^*}{L^*}, \]  

where \( w^* \) is the competitive foreign wage and \( s^* \) is the foreign firm’s subsidy level.

The domestic firm’s local plant and the union conduct an efficient Nash bargain to jointly determine \( w \) and \( L \). However, if no agreement is reached between the union and the domestic firm, the domestic firm can produce abroad in the fourth country and pay workers the competitive wage \( \xi \) that prevails in the fourth country. Thus, the reservation profit of the firm is:

\[ \Psi = (P - \xi)L, \]  

where \( L \) is the Cournot output level of production of the domestic firm when its production facilities are located abroad and \( P(L + L^*) \) is the home country’s price. The reservation profit \( \psi = P - \xi \) \( L \) must be less than domestic profit \( \Pi = P(L + L^*) - w + s \) \( L \), otherwise the domestic firm will produce abroad.

We assume here that \( w - s < w^* \), otherwise the home country will switch its production abroad².

Let \( \pi = \Pi - \psi \). Thus following Mezzetti & Dinopoulos (1991), the generalized Nash bargaining problem is:

\[ \text{Max}_{x \in \mathcal{X}} \{ U^x \} = \text{Max}_{x \in \mathcal{X}} \{ ((w - b)^\beta L^x) \} \]  

where \( \alpha \) is the relative bargaining power of the union.

The first order conditions with respect to \( w \) and \( L \) are:

\[ \alpha \pi U_w + (1-\alpha) \ U \pi_w = 0, \]  

\[ \alpha \pi U_L + (1-\alpha) \ U \pi_L = 0. \]  

¹ Mezzetti & Dinopoulos (1991) and Brander & Spencer (1988)

² Pencavel (1984)
\[ \alpha \pi U_w + (1-\alpha) U_p, \quad (9) \]

\[ \alpha \pi = (\alpha - 1) L \pi, \quad (10) \]

Equation (10) represents the Nash Bargaining Curve (Mezzetti & Dinopoulos, 1991).

Dividing equation (7) by (8) we obtain:

\[ \frac{U_w}{U_p} = \frac{\pi_w}{\pi_p}. \quad (11) \]

Equation (11) implicitly defines \( w \) as a function of \( L \), given \( L^* \), \( s \) and \( s^* \). Equation (11) defines the Efficient Contract. The Nash Bargaining Curve (equation (10)) together with the Efficient Contract (equation (11)) determines the Nash Bargaining Equilibrium.

Using equations (3), (4), (5) and (11) we can obtain the following representation of the Efficient Contract:

\[ \frac{w - b}{\beta} = w - P - P' L - s. \quad (12) \]

If the union is wage-neutral (\( \beta = 1 \)), then equation (12) becomes independent of \( w \) and the domestic employment level will not depend on the domestic wage rate.

Using equations (3), (4), (5) and (10) we can obtain the following representation of the Nash Bargaining Curve:

\[ w = P' L + P + s - \alpha P' - \frac{s}{L}. \quad (13) \]

The nonunionized foreign firm faces a given wage \( w^* \) and its subsidy inclusive profit is:

\[ \Pi^* = \{P(L + L^*) - w^* + s\} L^* \quad (14) \]

where \( s^* \) is the foreign firm’s subsidy level. The first order condition of the foreign firm under the Cournot-Nash behavior is

\[ P + L^* P' = w^* - s^*, \quad (15) \]

which states that marginal revenue equals the foreign wage \( w^* \), minus the foreign subsidy \( s^* \). Equation (15) together with the Nash Bargaining Curve (equation (13)) and the Efficient Contract (equation (12)) constitute a system of three equations in three unknowns whose solution is the equilibrium outcome of the model. Mezetti & Dinopoulos (1991) showed that a unique equilibrium exists in such a model.

**National Welfare**

Let \( N \) be the total domestic workforce. Domestic workers who cannot find employment in the firm are absorbed by the competitive nonunionized sector and paid a competitive wage \( b < w \). Then the number of workers employed by the competitive sector equals \((N-L)\). That allows the competitive sector to produce the numeraire good with the following technology:

\[ X = b(N-L) \quad (16) \]

Now, let us consider a utilitarian representation of domestic welfare where we add the profit, wage incomes, and the tax burden throughout the economy:

\[ W = (P - w + s)L + wL + b(N - L) = P L + c(N - L) = (P - b)L + bN. \quad (17) \]

Such a welfare function is common to this literature (Bandyopadhyay & Bandyopadhyay, 2001; Brander & Spencer, 1988).

National Welfare in the foreign country (where every worker receives a competitive wage \( w^* \)) has a similar structure and is given by

\[ W^* = (P - w^* + s^*)L^* + w^* N^* - s^* L^* = (P - w^*)L^* + w^* N^*. \quad (18) \]

**Comparative Statics**

In this section, I examine the comparative statics effects of a union’s relative bargaining power, domestic and foreign subsidies, and the foreign wage rate. Following Brander and Spencer (1985), I assume that the domestic firm’s marginal revenue declines in the foreign output level and the foreign firm’s marginal revenue declines in the domestic output level. In other words:

\[ P' + L P'' < 0, \quad \text{and} \quad P' + L^* P'' < 0 \quad (18a) \]

Equations (12), (13) and (15) determine the three endogenous variables of the model. Assuming that the union is wage-neutral and that

\[ (1 - \alpha) P' + \frac{\psi}{L^*} < 0, \quad (19a) \]

we receive the following results (see Appendix D):

\[ \frac{dw}{d\alpha} > 0, \quad \frac{dU}{d\alpha} > 0, \quad \frac{dL}{d\alpha} = \frac{dL^*}{d\alpha} = 0, \quad \text{and} \]

see the summary below.

**Proposition 1.** If the union is wage-neutral, then an increase in its relative bargaining power
i. increases the domestic wage.  
ii. does not affect both domestic and foreign employment (output) level and welfare.  
iii. increases the union’s utility level and decreases the domestic firm’s profit level.
Assuming that the union is wage-neutral and the inverse demand function is convex, (and utilizing the same method as in Appendix D), it will yield the results below:

Proposition 2. If the union is wage neutral, then an increase in foreign wage
i. increases domestic employment (output) level and decreases foreign employment (output) level.
ii. increases domestic welfare, union’s utility level, and has indeterminate effects on foreign welfare, domestic profits, and the union’s utility.
iii. increases domestic profits and the union’s utility regardless of the union’s wage orientation if the union has almost no bargaining power.
iv. increases domestic profits if the union has almost all the bargaining power.

Proposition 3. If the union is wage neutral, then an increase in foreign subsidy
i. decreases domestic welfare and employment (output) level and increases foreign employment (output) level.
ii. decreases domestic profits regardless of union’s wage orientation if the union has almost no bargaining power.

Proposition 4. If the union is wage neutral, then an increase in domestic subsidy
i. increases domestic employment (output) level; decreases foreign employment (output) level.
ii. has an ambiguous effect on domestic welfare and decreases foreign welfare.
iii. increases the union’s utility level regardless of the union’s wage orientation if the union has almost no bargaining power.

It follows from Appendix B that if the union is employment-oriented and has almost all the bargaining power, then an increase in foreign subsidy also decreases domestic employment level and increases foreign employment levels. From Appendix A we see that if the union has almost no bargaining power then regardless of its wage orientation an increase in foreign subsidy decreases domestic employment and increases foreign employment.

Mezzetti & Dinopoulos (1991) abstracted their model from unionized duopolies in exporting industries. They conjectured that an increase in the export subsidy would increase employment level and therefore would be welcomed by labor unions. We see from the propositions above that an increase in foreign export subsidy does increase foreign employment level in the case of a wage neutral union.

Finally, assuming that the domestic firm can credibly threaten to shift production abroad (and is utilizing the same method as in Appendix D), it will yield the proposition below.

Proposition 5. If the domestic firm can credibly threaten to shift production abroad, then an increase in the outsourcing country’s wage
i. increases the utility level of the union, decreases domestic profits, does not affect domestic and foreign employment and welfare if the union is wage-neutral.
ii. increases the utility of the union, increases domestic employment and welfare, decreases foreign employment, and has an ambiguous effect on domestic profits and foreign welfare if the union is labor-oriented.

We see from the proposition above that an increase in the outsourcing country’s wage may have unexpected welfare effects if the union is labor-oriented.

Summary and Conclusions

In this paper I developed a simple static model to understand unionization and international competition. I used the model to inquire into the effects of bargaining power, subsidies, and the threat to shift production abroad. I found that if the domestic firm is a multinational that can credibly threaten to shift production abroad, then an increase in the outsourcing country’s wage may decrease the union’s utility level, and decrease the domestic country’s welfare if the union is labor-oriented.

My analysis highlights the need to empirically estimate the employment and wage orientation of a union. Also, an interesting implication is that an increase in domestic subsidy increases the union’s utility level regardless of whether the union has almost no bargaining power. If the inverse demand function is convex, then an increase in foreign wage increases the utility level of a wage-neutral union. If the union has almost no bargaining power, then an increase in foreign wage increases domestic profits and the union’s utility level regardless of the union’s wage-orientation and the shape of the inverse demand function. If the union is wage-neutral, then an increase in domestic subsidy increases the union’s utility level even if the union has almost no bargaining power.

Finally, observe that my analysis is abstracted from modeling a duopolistic competition between two unionized exporting firms. This presents an interesting avenue for future research.

Table 1

<table>
<thead>
<tr>
<th>Summary of Results</th>
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<tbody>
<tr>
<td>Wage Neutral Union</td>
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<td>( \alpha \uparrow )</td>
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<td>( L^* ) No change ( \downarrow \leftrightarrow \leftrightarrow \leftrightarrow \leftrightarrow \uparrow \leftrightarrow \downarrow \leftrightarrow \leftrightarrow \leftrightarrow \leftrightarrow \uparrow \leftrightarrow \leftrightarrow</td>
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<tr>
<td>( W ) No change ( \leftrightarrow \leftrightarrow \leftrightarrow \leftrightarrow \uparrow \leftrightarrow \leftrightarrow \leftrightarrow \leftrightarrow \uparrow \leftrightarrow \leftrightarrow</td>
</tr>
<tr>
<td>( W^* ) No change ( \downarrow \leftrightarrow \leftrightarrow \leftrightarrow \leftrightarrow \uparrow \leftrightarrow \leftrightarrow \leftrightarrow \leftrightarrow \uparrow \leftrightarrow \leftrightarrow</td>
</tr>
</tbody>
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Note: \( \leftrightarrow \) means that the change in the variable is indeterminate.
References


Footnotes

1  Mezetti & Dinopoulous (1991) and Brander & Spencer (1988) find that if the union is employment-oriented, then an increase in its bargaining power increases domestic welfare.

2  In other words, I assume that exogenous variables are such that the threat of outsourcing is a threat that is never realized. See Appendix C for additional information.

3  Table1 summarizes the results of this section.

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Appendix A

Comparative Static Effects for a Union with Almost No Bargaining Power

If the union has almost no bargaining power $\alpha = 0$, then it follows from system (19) that

$$
\begin{pmatrix}
\frac{1}{\beta} & -1 \\
0 & (P' + LP'') \\
-1 & (LP'' + 2P')
\end{pmatrix}
\begin{pmatrix}
\frac{dw}{dw^*} \\
\frac{dl}{dw^*} \\
\frac{dl'}{ds^*}
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix},
$$

(A.1)

$$
\begin{pmatrix}
\frac{1}{\beta} & -1 \\
0 & (P' + LP'') \\
-1 & (LP'' + 2P')
\end{pmatrix}
\begin{pmatrix}
\frac{dw}{ds^*} \\
\frac{dl}{ds^*} \\
\frac{dl'}{ds^*}
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
-1 \\
0
\end{pmatrix},
$$

(A.2)

$$
\begin{pmatrix}
\frac{1}{\beta} & -1 \\
0 & (P' + LP'') \\
-1 & (LP'' + 2P')
\end{pmatrix}
\begin{pmatrix}
\frac{dw}{ds} \\
\frac{dl}{ds} \\
\frac{dl'}{ds}
\end{pmatrix}
= 
\begin{pmatrix}
-1 \\
0 \\
-1
\end{pmatrix},
$$

(A.3)

It follows from systems (A.1), (A.2) and (A.3) that

$$
\frac{dw}{dw^*} = \frac{dw}{ds^*} = \frac{dw}{ds} = 0,
$$

(A.4)

$$
\frac{dl}{dw^*} = \frac{1}{\beta} \frac{(P' + LP'')}{|A|} > 0.
$$

(A.5)

$$
\frac{dl}{ds^*} = -\frac{1}{\beta} \frac{(P' + LP'')}{|A|} < 0.
$$

(A.6)

$$
\frac{dl}{ds} = \frac{1}{\beta} \frac{(2P' + LP'')}{|A|} > 0.
$$

(A.7)

$$
\frac{dl'}{dw^*} = -\frac{1}{\beta} \frac{(2P' + LP'')}{|A|} < 0.
$$

(A.8)

$$
\frac{dl'}{ds^*} = \frac{1}{\beta} \frac{(2P' + LP'')}{|A|} > 0.
$$

(A.9)
\[
\frac{dL^*}{ds} = -\frac{1}{\beta} \left( P^* L^* P'' \right) < 0. \tag{A.10}
\]

\[
\frac{d(L + L')}{dw} = -\frac{1}{\beta} \frac{P'}{|A|} < 0. \tag{A.11}
\]

\[
\frac{d(L + L')}{ds} = \frac{1}{\beta} \frac{P'}{|A|} > 0. \tag{A.12}
\]

\[
\frac{d(L + L')}{ds} = \frac{1}{\beta} \frac{P'}{|A|} > 0. \tag{A.13}
\]

It follows from the equations above that domestic profits and domestic welfare decrease with foreign subsidy and increase with foreign wage regardless of the union’s wage orientation.
Appendix B

Comparative Static Effects for a Union with Almost All the Bargaining Power

If the union has almost all bargaining power (\(\alpha = 1\)), then it follows from system (19) that

\[
\begin{pmatrix}
\frac{1}{\beta} - 1 & 2P' + LP'' & P' + LP'' \\
0 & P' + LP'' & 2P' + LP'' \\
-1 & P' + \frac{\psi}{L^2} & P'
\end{pmatrix}
\begin{pmatrix}
\frac{dw}{dw^*} \\
\frac{dl}{dw^*} \\
\frac{dl'}{dw^*}
\end{pmatrix}
= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},
\]

(B.1)

\[
\begin{pmatrix}
\frac{1}{\beta} - 1 & 2P' + LP'' & P' + LP'' \\
0 & P' + LP'' & 2P' + LP'' \\
-1 & P' + \frac{\psi}{L^2} & P'
\end{pmatrix}
\begin{pmatrix}
\frac{dw}{ds} \\
\frac{dl}{ds} \\
\frac{dl'}{ds}
\end{pmatrix}
= \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix},
\]

(B.2)

\[
\begin{pmatrix}
\frac{1}{\beta} - 1 & 2P' + LP'' & P' + LP'' \\
0 & P' + LP'' & 2P' + LP'' \\
-1 & P' + \frac{\psi}{L^2} & P'
\end{pmatrix}
\begin{pmatrix}
\frac{dw}{ds} \\
\frac{dl}{ds} \\
\frac{dl'}{ds}
\end{pmatrix}
= \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix},
\]

(B.3)

It follows from systems (B.1), (B.2) and (B.3) that:

\[
\frac{dw}{dw^*} = \frac{\left(\frac{1}{\beta} P' + LP''\right) - \frac{\psi}{L^2} + P'^2}{|A|} < 0,
\]

(B.4)

\[
\frac{dl}{dw^*} = \frac{1}{|A|} > 0, \quad \text{if} \quad 0 < \beta \leq 1.
\]

(B.5)

\[
\frac{dl'}{dw^*} = \left(\frac{1}{\beta} - 1\right)\left(P' + \frac{\psi}{L^2}\right) + (2P' + LP'')\frac{1}{|A|} < 0, \quad \text{if} \quad 0 < \beta \leq 1
\]

(B.6)

\[
\frac{dl}{ds^*} = -\left(\frac{1}{\beta} P' + LP''\right)\frac{1}{|A|} < 0, \quad \text{if} \quad 0 < \beta \leq 1.
\]

(B.7)

\[
\frac{dl'}{ds^*} = \left(\frac{1}{\beta} - 1\right)\left(P' + \frac{\psi}{L^2}\right) + (2P' + LP'')\frac{1}{|A|} > 0, \quad \text{if} \quad 0 < \beta \leq 1.
\]

(B.8)
Appendix C

A Reservation Profit Example

As an example let us consider a wage-neutral union and let the inverse demand function for the export good be:

\[ P = P(Q) = a - (L + L^*), \]

where \( Q \) is the market output.

Then, it follows from equations (12), (13) and (15) that:

\[ \Omega = w - s - w^* = \frac{A(\alpha - 2)}{3} - a - 2s - w^* - \frac{2A}{3} - \frac{3\alpha \psi}{A}, \]

where \( A = w^* - s + a - 2b + 2s. \)

Here we observe that \( \Omega \) is a function of exogenous variables only. Further, we note that if \( \Omega < 0 \), then the domestic firm will not switch its production abroad.
Appendix D

Comparative Static Effects for a Wage-neutral Union

With totally differentiating equations (12), (13) and (15) we can calculate the changes in $L$, $L^*$, and $w$ with respect to small changes in, $a$, $s$, $s^*$ and $w^*$.

\[
\begin{align*}
\left( \frac{1}{\beta} - 1 \right) dw + (2P' + LP') dL + (P' + LP') dL^* &= -ds, \\
(0) dw + (P' + L^*P^*) dL + (2P' + L^*P^*) dL^* &= dw^* - ds^*, \\
(-1) dw + \left( LP^* + 2P' - \alpha LP^* - \alpha P' + \psi \frac{\alpha}{L^*} \right) dL + (P' + P^* L - \alpha LP^*) dL^* &= \\
= (P' L + \frac{\psi}{L}) d\alpha - ds.
\end{align*}
\]

(D.1)

Now, let us assume that:

and that the union is wage-neutral ($\beta = 1$).

Let $A$ be the coefficient matrix. Then,

\[
|A| = (-1) \left[ (2P' + LP^*) (2P' + L^*P^*) - (P' + LP^*) (P' + L^*P^*) \right]
\]

(D.4)

Since by (18a) we have that $(P' + LP^*) < 0$ and, $(P' + L^*P^*) - (P' + LP^*) < 0$ then $|A| < 0$, and matrix $A$ is nonsingular. Using Cramer’s rule we determine that:

\[
\begin{align*}
\frac{dw}{d\alpha} &= -\left( P' L + \frac{\psi}{L} \right), \\
\frac{dL}{d\alpha} &= 0, \text{ and } \frac{dL^*}{d\alpha} = 0.
\end{align*}
\]

(D.5)

(D.6)

It follows from equation (D.3) that $\left( P' L + \frac{\psi}{L} \right) < 0$, which in turn implies that $\frac{dw}{d\alpha} > 0$. In other words, we observe that an increase in the relative bargaining power of the union increases the wage rate of the domestic workers if the union is wage-neutral.
Totally differentiating the union’s utility function and considering that the union is wage neutral ($\beta=1$), we obtain

\[
\frac{dU}{d\alpha} = L \frac{dw}{d\alpha} + (w - b) \frac{dL}{d\alpha}.
\]  
(D.7)

Since $\frac{dw}{d\alpha} > 0$ and $\frac{dL}{d\alpha} = 0$, and it follows that an increase in the relative bargaining power of the union increases the utility level of a wage-neutral union.