Journal of CENTRUM Cathedra • Volume 4, Issue 2, 2011 • 197-215

JCC
Journal of
CENTRUM
Cathedra

CENTRUM
CENTRO DE NEGOCIOS

Avoiding Large Differences in Weights in Cross-Efficiency Evaluations: Application to the Ranking of Basketball Players

William W. Cooper
The Red McCombs School of Business, The University of Texas at Austin,Austin, USA

Nuria Ramón
Centro de Investigación Operativa, Universidad Miguel Hernández, Spain

José L. Ruiz *
Centro de Investigación Operativa, Universidad Miguel Hernández, Spain

Inmaculada Sirvent
Centro de Investigación Operativa, Universidad Miguel Hernández, Spain

Abstract

Because of data envelopment analysis (DEA) flexibility in the choice of weights, assessment of decision-making units (DMUs) often involves weighting only a few inputs and outputs and ignoring the remaining variables by assigning them a zero weight. Widespread literature indicates the need to avoid zero weights, and some authors claim that the fact that a given DMU attaches very different weights to the variables involved in the assessments may be a concern (see, for example, Cooper, Seiford, & Tone, 2007). The aim of this paper was to prevent unrealistic weighting schemes in cross-efficiency evaluations through an extension of the multiplier bound approach (Ramón, Ruiz, & Sirvent, 2010a) based on "model" DMUs. The approach in that paper guarantees nonzero weights while at the same time it tries to avoid large differences in the values of multipliers. An application to the ranking of basketball players involved specifying a limit for allowable differences in the relative importance that players attach to different aspects of the game by reflecting those observed in the weight profiles of some model players, which are selected according to expert opinion. The approach provided results that are consistent with basketball expert opinion and illustrated why the classical approaches to cross-efficiency evaluation, which include the benevolent and aggressive formulations, may lead to unreasonable results.

Keywords: Cross-efficiency evaluation, DEA, ranking, weights, basketball

JEL Classification code: C61

Cross-efficiency evaluation, as introduced in Sexton, Silkman, and Hogan (1986) and Doyle and Green (1994a), is an extension of the original data envelopment analysis (DEA) methodology (Charnes, Cooper, & Rhodes, 1978) aimed at providing a ranking of the decision-making units (DMUs). DEA weights are unit-specific, so they provide a self-evaluation of the DMUs that does not allow for derivation of an ordering. The basic idea of cross-efficiency evaluation is to assess each unit with the weights of all the DMUs instead of with its own weights alone. Calculating the cross-efficiency score of a given unit usually involves taking the average of its cross-efficiencies, obtained using the profiles of weights provided by all the DMUs. Thus, evaluation of each unit occurs within the range of weights of all the DMUs to provide a peer evaluation (instead of a self-evaluation), allowing for ranking of the DMUs.

To address the problems associated with alternate optima for the weights in the CCR model, some authors have proposed the use of alternative secondary goals to the choice of the profiles of weights for cross-efficiency evaluations. Such models would include the well-known benevolent and aggressive formulations (Doyle & Green, 1994a; Sexton et al., 1986), which seek to globally maximize or minimize, respectively, the cross-efficiencies of all the DMUs, while maintaining the self-evaluation of the unit under assessment (see Liang, Wu, Cook, & Zhu, 2008a, for extensions of these models). Wang and Chin (2010) put forward a different approach, a neutral DEA model through which each DMU determines the weights only from its own point of view without considering its effects on other DMUs. Each DMU chooses weights to maximize the relative contribution of the outputs in its assessment, which can effectively reduce the number of zero weights for outputs.

The approach proposed in the current paper, like that in Ramón, Ruiz, and Sirvent (2010b), is in accordance with Wang and Chin's (2010) model in the sense that each DMU makes its own choice of weights without considering the effects on other DMUs. In this paper, each DMU chooses its profile of weights attempting to avoid large differences in the weights attached to both inputs and outputs, which also guarantees nonzero weights. In Ramón, Ruiz, and Sirvent (2011), the differences in the weights that the different DMUs attach to the same variable are reduced. See also Liang, Wu, Cook, and Zhu (2008b) and Wu, Liang, and Chen (2009) for examples of approaches from the perspective of the game theory.

Some researchers claim that cross-efficiency evaluation eliminates unrealistic weighting schemes without the need to elicit weight restrictions. The idea is that the amalgamation of weights in the cross-efficiency evaluation cancels out unrealistic weighting schemes (Anderson, Hollingsworth, & Inman, 2002; Doyle & Green, 1994a; Sexton et al., 1986). The idea is sound, but avoiding unreasonable weights instead of expecting elimination of their effects in the summary of the cross-efficiency evaluation may lead to more comprehensive cross-efficiency scores. Thus, the focus of the cross-efficiency evaluation approach proposed in this paper is on the weight profile choice of each DMU, in an attempt to prevent unrealistic weighting schemes. Specifically, the model involves extending the multiplier bound approach to the assessment of efficiency without slacks (Ramón et al., 2010a) for use in cross-efficiency evaluations, which ensures nonzero weights and aims to avoid large differences in weights.

Widespread literature indicates the need to avoid zero weights in DEA assessments, and a number of authors emphasize the importance of exercising some control over the variation in factor weights resulting from DEA flexibility (Roll & Golany, 1993). Cooper, Seiford, and Tone (2007) stated that large differences in the weights that a given DMU attaches to the different inputs and outputs may be a concern (in extreme cases, when zero weights exist, the corresponding variables are ignored in the analysis). Cook and Seiford (2008) noted, "The AR [assurance region] concept was developed to prohibit large differences in the values of multipliers" (p. 8). The approach in Ramón et al. (2010a) relates to the assessment of the DMUs having nonzero slacks in their optimal solutions in the CCR model (i.e., those in $F \cup NF^1$), evaluated after specifying a limit for the allowable differences in weights determined by using the optimal solutions of the extreme efficient units (the DMUs in E) with the least dissimilar weights. The approach also guarantees nonzero weights.

A variant of this approach is evident in the current paper, because the limit for the allowable differences in weights is specified in an identical manner, but taking consideration of the differences observed in the weight profiles of some model DMUs instead of those in the extreme efficient DMUs. The notion of *model DMUs* is apparent in the literature: Charnes, Cooper, Huang, and Sun (1990) and Brockett, Charnes, Cooper, Huang, and Sun (1997) used the optimal weights of some model (good) DMUs, selected based on expert opinion, in cone-ratio models to define some admissible directions. In the current paper, expert opinion aids in identifying the model DMUs, and their weights are considered in setting the limit for allowable differences in the weights used in the assessment of the DMUs.

Ramón et al. (2010a) assessed the DMUs in $F \cup NF$ with weights that are different from those provided by the CCR model for these DMUs, which is of particular interest in the attempt to prevent unreasonable weights in cross-efficiency evaluations evident in this paper. Detailed analysis of the choice of weights that inefficient DMUs make is necessary to address the gap in cross-efficiency evaluation literature. In practice, many of the inefficient DMUs have zeros in their optimal solutions for the weights in the CCR model, and these solutions are generally unique. Due to this uniqueness, these optimal solutions will be obviously the profiles of weights that the inefficient DMUs provide for the calculation of cross-efficiencies when using an approach based on a choice of weights among alternate optima, regardless of the criterion of selection. The use of the profiles of weights provided by the multiplier bound approach (Ramón et al., 2010a) helps to overcome this difficulty because the procedure does not involve selecting between the optimal solutions of the DMUs in $F \cup NF$. Further discussion is evident in the context of the application in this paper, through which the undesired effects of using the profiles of weights provided by the CCR model for the DMUs in $F \cup NF$ in the cross-efficiency evaluation are presented.

The current paper includes an application of the proposed approach to the ranking of basketball players. Cooper, Ruiz, and Sirvent (2009) reported the results of an application of DEA to the assessment of the effectiveness of basketball players in the context of the Spanish ACB League (the premier league in Spain). The focus was on the multipliers, with the purpose of providing component profiles to allow for identification of strengths and weaknesses of individual players. Using DEA, Cooper et al. tried to avoid the difficulties evident in the ACB index of player assessment (same assessment used in other competitions, such as the Euroleague), which attaches the same value to all the statistical indicators (e.g., points, rebounds, assists, fouls, etc.). The idea was to take advantage of DEA flexibility in the choice of weights, and based on some information from the technical staff of an ACB team Cooper et al. could fortunately access, incorporating the opinion of experts on the relative importance of the different aspects of the game in the form of some assurance region (AR) constraints (Thompson, Singleton, Thrall, & Smith, 1986). Eventually, Cooper et al. (2009) used the two-step procedure in Cooper, Ruiz, and Sirvent (2007) to make a specific choice of weights between the alternate optima. However, the analysis in Cooper et al. (2009) did not address the ranking of players, which is an aspect of great interest in assessments in such a context.

The purpose of this paper was to make a cross-efficiency evaluation of basketball players of the ACB League to rank the players. ACB League information allowed for identification of some model players through the most valuable player (MVP) award. These players were necessary to set the limit for the allowable differences in the importance attached to the different aspects of the game in the assessment of players. The proposed approach yielded results that are consistent with basketball expert opinion.

Cross-efficiency evaluation has already been used in applications to sports. Wu, Liang, and Yang (2009) measured the performance of nations in the Summer Olympic Games, and Wu, Liang, and Chen (2009) applied a DEA game cross-efficiency approach to Olympic rankings. For other applications of cross-efficiency evaluation, see Sexton et al. (1986) on nursing homes; Oral, Kattani, and Lang (1991) on R&D projects; Doyle and Green (1994b) on higher education; Green, Doyle, and Cook (1996) on preference voting; T. Y. Chen (2002) on the electricity distribution sector; and Lu and Lo (2007) on economic environmental performance.

DEA has been useful in evaluating basketball players (Cooper et al., 2009), baseball players (Anderson, 2004; Anderson & Sharp, 1997; Chen & Johnson, 2010; Sexton & Lewis, 2003; Sueyoshi, Ohnishi, & Kinase, 1999), golfers (Fried, Lambrinos, & Tyner, 2004; Fried & Tauer, 2011), and football players (Alp, 2006). In football, researchers have applied the methodology from the point of view of soccer teams (Boscá, Liern, Martínez, & Sala, 2009; Espitia-Escuer & García-Cebrián, 2004; González-Gómez & Picazo-Tadeo, 2010; Haas, 2003), coaches (Dawson, Dobson, & Gerrard, 2000), and clubs (Barros, Assaf, & Sá-Earp, 2010; Barros & Leach, 2006). Researchers have further measured relative efficiency in sports at country level with DEA models, in particular assessing the performance of participating nations in the Summer Olympics (Lozano, Villa, Guerrero, & Cortés, 2002; Wu, Zhou, & Liang, 2010; Zhang, Li, Meng, & Liu, 2009). Finally, applications of DEA to analyze efficiency in sports from other perspectives are apparent. Fizel and D'Itri (1999) studied the impact of practices, such as firing and hiring managers, on organizational performance, Volz (2009) provided efficiency scores of not only team performance but also player salaries in Major League Baseball, and Einolf (2004) measured franchise payroll efficiency in the National Football League and Major League Baseball.

The following section addresses the developments corresponding to the cross-efficiency evaluation for the case of using model DMUs as an extension of the approach in Ramón et al. (2010a). Next, the results of the application are reported. The final section indicates the conclusions of the research.

Theoretical Aspects: The Cross-Efficiency Evaluation Approach

The research involved using a variant of the basic approach in Ramón et al. (2010a) based on the use of model DMUs to choose the weight profiles for cross-efficiency evaluation (this is actually mentioned as a possible extension in that paper). A discussion of the developments corresponding to the variant approach, including both the models used and their properties, follows. The basic approach consists of a two-step procedure: (a) specify the weight bounds to determine a limit for the allowable differences in weights by selecting weights from the alternate optima of the extreme efficient units, trying to avoid large differences between multipliers, and (b) incorporate the bounds into the DEA formulations used to assess the efficiency of inefficient units. Following these ideas, the models that provide the weight profiles that the DMUs use in calculating the cross-efficiencies in the present paper are therefore different depending on whether or not the corresponding DMU is a model DMU.

Model DMUs

Throughout this section, the assumption is that n DMUs use m inputs to produce s outputs, which must be strictly positive. M is the set of model DMUs, and DMU_d is a given unit in M. Model DMUs are assumed to be Pareto-efficient units, which usually have alternative optimal solutions for their weights in the CCR model. In such a situation, model DMU_d must choose, among all of its alternate optima in the CCR model, the optimal solution of the following model as the profile of weights for use in the cross-efficiency calculation:

$$\text{Max } \varphi_d \tag{1}$$

s.t.

$$\sum_{i=1}^{m} \nu_i^d x_{id} = 1, \tag{1.1}$$

$$\sum_{r=1}^{s} \mu_r^d y_{rd} = 1, \tag{1.2}$$

$$-\sum_{i=1}^{m} \nu_i^d x_{ij} + \sum_{r=1}^{s} \mu_r^d y_{rj} \le 0, \; j = 1,\ldots,n, \tag{1.3}$$

$$z_I \le \nu_i^d x_{id} \le h_I, \; i = 1,\ldots,m, \tag{1.4}$$

$$z_O \le \mu_r^d y_{rd} \le h_O, \; r = 1,\ldots,s, \tag{1.5}$$

$$\frac{z_I}{h_I} \ge \varphi_d, \tag{1.6}$$

$$\frac{z_O}{h_O} \ge \varphi_d \tag{1.7}$$

$$\nu_i^d, \mu_r^d, z_I, h_I, z_O, h_O, \varphi_d \ge 0.$$

Model 1 corresponds to the first step of the procedure in Ramón et al. (2010a). The idea of Model 1 is the following: Constraints 1.1 to 1.3 allow for all the optimal multipliers in the CCR model for DMU_d. In 1.4 to 1.5, the constraints force all the input virtuals and all the output virtuals to vary in between the bounds z_I and h_I and the bounds z_O and h_O, respectively. As a result, the ratios between each couple of input (output) virtuals are all greater than or equal to z_I/h_I (z_O/h_O). Model 1 involves maximizing φ_d, and both z_I/h_I and z_O/h_O are greater than or equal to φ_d (as a result of 1.6 and 1.7). Thus, the aim is to maximize the minimum of the two ratios and, in this sense, look for the weight profiles with the least dissimilar virtual inputs and outputs to allow DMU_d to be rated as efficient.

Proposition 1. Model 1 has a global optimum.

Proof. The purpose of the proof is to show that the search for the global optimum of Model 1 can be confined to a compact subset of the feasible region of Model 1, which ensures the existence of such a global optimum because we would be maximizing a continuous function in a compact set. Because the feasible region of Model 1 is a closed set and all its variables except h_I and h_O are bounded, finding this compact set involves simply bounding these two variables appropriately.

Let $\left(\tilde{v}_1^d,...,\tilde{v}_m^d,\tilde{\mu}_1^d,...,\tilde{\mu}_s^d\right)$ be an optimal solution of the dual formulation of the CCR model satisfying $\tilde{v}_i^d > 0$, i = 1,…,m, and $\tilde{\mu}_r^d > 0$, r = 1,…,s (such an optimal solution exists because DMU_d is Pareto-efficient). Next, define $\tilde{z}_I := \min_{i=1,...,m}\left\{\tilde{v}_i^d x_{id}\right\}$, $\tilde{h}_I := \max_{i=1,...,m}\left\{\tilde{v}_i^d x_{id}\right\}$, $\tilde{z}_O := \min_{r=1,...,s}\left\{\tilde{\mu}_r^d y_{rd}\right\}$, $\tilde{h}_O := \max_{r=1,...,s}\left\{\tilde{\mu}_r^d y_{rd}\right\}$, and $\tilde{\varphi}_d := \min\left\{\frac{\tilde{z}_I}{\tilde{h}_I},\frac{\tilde{z}_O}{\tilde{h}_O}\right\}$, which are all strictly positive scalars (because the assumption is that actual inputs and outputs are strictly positive). It is easy to check that $\left(\tilde{v}_1^d,...,\tilde{v}_m^d,\tilde{\mu}_1^d,...,\tilde{\mu}_s^d,\tilde{z}_I,\tilde{h}_I,\tilde{z}_O,\tilde{h}_O,\tilde{\varphi}_d\right)$ is a feasible solution of Model 1 with an associated value in the objective equals $\tilde{\varphi}_d$. In addition, if $\left(v_1^d,...,v_m^d,\mu_1^d,...,\mu_s^d,z_I,h_I,z_O,h_O,\varphi_d\right)$ is a feasible solution of Model 1 with either $h_I > \frac{1}{\tilde{\varphi}_d}$ or $h_O > \frac{1}{\tilde{\varphi}_d}$, then $\varphi_d < \tilde{\varphi}_d$ (if, for example, $h_I > \frac{1}{\tilde{\varphi}_d}$, then $\varphi_d \le \frac{z_I}{h_I} \le \frac{1}{h_I} < \tilde{\varphi}_d$). Thus, solving Model 1 involves restricting the search for its optimal solution to those feasible solutions satisfying the two additional restrictions, $h_I \le \frac{1}{\tilde{\varphi}_d}$ and $h_O \le \frac{1}{\tilde{\varphi}_d}$. This subset of the feasible region of Model 1 is therefore a compact set, which ensures the existence of a global maximum.

To find the global optimum, one could use an approach based on a parametric linear programming (LP) problem where φ_d serves as the parameter. Because φ_d belongs to (0,1], one must solve Model 1 for a set of values for φ_d within that range. The idea is to search for the maximum value of the parameter for which a feasible solution of Model 1 exists. A simple search procedure is to start at the upper end of the range, φ_d = 1, and decrease the parameter, with a decrement of, say, 0.0001, until a feasible solution emerges. Obviously, if solutions that are more accurate are required, one could choose a smaller decrement. Note, however, that in the case of having either one input or one output, one could convert Model 1 into a linear problem. For example, having only one output (as in the application presented later) would require maximization of z_O/h_O subject to 1.1 to 1.3 and 1.5 and, following Charnes and Cooper (1962), one could then determine the optimal solution by means of a linear problem.

Model 1 is formulated in terms of virtuals instead of absolute weights (this is also proposed as another possible extension of the basic approach in Ramón et al. (2010a)). The idea of the models used in the cross-evaluation approach proposed in this paper is to avoid large differences in the relative importance attached to the involved variables, which in many instances one can achieve more appropriately using virtuals instead of absolute weights. In particular, the indicators used in the application portion of this paper to describe the different aspects of basketball are measured in units such that the comparisons between the corresponding absolute weights might become meaningless (see Sarrico & Dyson, 2004, for a discussion on the use of virtual weights).

The following properties of Model 1 are of interest:

Proposition 2. $\varphi_d^* > 0$.

Proof. Because the DMU_d's in M are assumed to be Pareto-efficient points on the frontier, there exists at least one optimal solution of the dual multiplier formulation of the CCR model for these units with nonzero weights. This guarantees that $\varphi_d^* > 0$, because of the maximization in Model 1.

Furthermore, φ_d^* provides insight into how much model DMU_d needs to unbalance the relative importance attached to the different inputs and to the different outputs to be rated as efficient. As indicated earlier, $\varphi_d^* \in (0,1]$, and the lower the value of φ_d^*, the larger the differences in the relative importance attached to the variables considered. If, for example, $\varphi_d^* = 0.2$, model DMU_d cannot be rated as efficient with a set of weights in which the lowest virtual input is higher than 20% of the highest one and the lowest virtual output is higher than 20% of the highest one. On the contrary, if $\varphi_d^* = 1$, DMU_d could be rated as efficient even with a profile of weights with the same virtual for all the inputs and the same virtual for all the outputs. The latter case might indicate that DMU_d performed well on all the variables.

Corollary 1. If $(v_1^{d*},...,v_m^{d*},\mu_1^{d*},...,\mu_s^{d*})$ is an optimal solution of Model 1 for a given DMU_d in M, then $v_i^{d*} > 0$, i =1,…,m, and $\mu_r^{d*} > 0$, r = 1,…,s.

Proof. Trivial.

The corollary is an important result because it guarantees that the profiles provided by Model 1 for the model DMUs do not have zero weights.

The Remaining DMUs

Suppose that DMU_d is a given unit that does not belong to M. For such DMUs, one needs to choose the optimal solution of the following LP problem as the profile of weights for use in the calculation of the cross-efficiencies:

$$\text{Max} \sum_{r=1}^{s} \mu_r^d y_{rd} \quad (2)$$

s.t.

$$\sum_{i=1}^{m} v_i^d x_{id} = 1, \quad (2.1)$$

$$-\sum_{i=1}^{m} v_i^d x_{ij} + \sum_{r=1}^{s} \mu_r^d y_{rj} \le 0, \; j = 1,...,n, \quad (2.2)$$

$$z_I \le v_i^d x_{id} \le h_I, \; i = 1,...,m, \quad (2.3)$$

$$z_O \le \mu_r^d y_{rd} \le h_O, \; r = 1,...,s, \quad (2.4)$$

$$\frac{z_I}{h_I} \ge \varphi^*, \quad (2.5)$$

$$\frac{z_O}{h_O} \ge \varphi^*, \quad (2.6)$$

$$v_i^d, \mu_r^d, z_I, h_I, z_O, h_O \ge 0.$$

where φ^* is defined as the scalar $\varphi^* := \min_{j \in M} \varphi_j^*$.

Model 2 corresponds to the second step of the two-step procedure in Ramón et al. (2010a). The main difference is the replacement of the set E of extreme efficient DMUs with that of the model DMUs, M, in the calculation of the minimum φ^* (in addition, virtuals instead of absolute weights are required in Model 2). Thus, the model DMUs are now used to specify the limit for the allowable differences in virtuals in the assessment of the remaining units. To be specific, DMU_d is assessed with weights in which both the virtual inputs among themselves and the virtual outputs among themselves cannot be more dissimilar than those of the model DMU that needs to unbalance more its weights (as measured by φ^*) to be rated as efficient.

Corollary 2. $\varphi^* > 0$.

Proof. Trivial because of Proposition 2.

With Model 2, assessment of each nonmodel DMU occurs with a set of strictly positive weights. The approach aids in avoiding one of the main difficulties with the weights provided by the CCR model for most of the inefficient DMUs in cross-efficiency evaluation (see further discussion in the application portion of this paper).

In practice, many of the inefficient DMUs belong to $F \cup NF$. For these DMUs, the CCR model provides weights with zeros, and these optimal solutions are generally unique. Model 2 allows for reassessment of the efficiency of each of the DMUs in $F \cup NF$ with reference to a different set of weights, also used in the calculation of the cross-efficiencies, to prevent unrealistic weighting schemes in cross-efficiency evaluations. As for the DMUs that are not model DMUs, if these need to unbalance their weights more than does the model DMU with the most dissimilar weights (i.e., more than φ^*) to achieve their DEA efficiency rating, they will have a lower efficiency score (in particular, the efficient DMUs may become inefficient). Otherwise, they will maintain their efficiency assessment with the CCR model, and with Model 2, one will choose among their optimal weights satisfying 2.3 to 2.6.

The Cross-Efficiency Scores

Based on the choice of weights proposed above, next one would need to obtain the cross-efficiency scores. If one denotes by $(v_1^{d*},...,v_m^{d*},\mu_1^{d*},...,\mu_s^{d*})$ the optimal solution of either Model 1, if the corresponding DMU_d is in M, or Model 2, if DMU_d is not a model DMU, then the cross-efficiency of a given DMU_j using the profile of weights provided by DMU_d is obtained as follows:

$$E_{dj} = \frac{\sum_{r=1}^{s} \mu_r^{d*} y_{rj}}{\sum_{i=1}^{m} v_i^{d*} x_{ij}} \tag{3}$$

Therefore, the cross-efficiency score of DMU_j is the average of these cross-efficiencies (i.e., the average of the column corresponding to DMU_j in the matrix of cross-efficiencies):

$$\overline{E}_j = \frac{1}{n}\sum_{d=1}^{n} E_{dj}, \quad j = 1,...,n. \tag{4}$$

In summary, the proposal was to extend the multiplier bound approach (Ramón et al., 2010a) for use in cross-efficiency evaluations in an attempt to prevent unrealistic weighting schemes because this approach guarantees nonzero weights while attempting to avoid profiles with large differences in weights. Incorporating prior information or expert opinion on the relative importance of the variables into the analysis (e.g., through AR constraints) often leads to nonzero weights and may further help reduce the dispersion in weights. Ramón et al. (2010a) aimed at avoiding large differences in weights without requiring prior information and so relied on that provided by the data. To implement such approach, one eventually needs to specify the value of φ^*, which represents the limit for the allowable differences in weights in the assessment of the DMUs. As for this specification, the optimal weights of the extreme efficient DMUs are used to that end, and φ^* is defined as the minimum of the ratios between the minimum and the maximum weights in the optimal solutions of the DMUs in E. The route followed in the approach to cross-efficiency evaluation in Ramón et al. (2010b) is very similar because they specify φ^* by considering all the DMUs assessed without slacks (or, equivalently, those that have nonzero weights). In the following application, available expert opinion allowed for identification of some good DMUs or players (termed model DMUs according to Charnes et al., 1990, and Brockett et al., 1997) whose weights are used to set the value of φ^* in an identical manner by simply replacing the set E, or that of the DMUs assessed without slacks, with M.

Application: Ranking Basketball Players

Data and Model Specification

The data relevant in this application are ACB official statistics from the ACB League website (http://www.acb.com/) corresponding to the 2008-2009 season. For players to appear in ACB statistics, they must have played at least two thirds of the games in the regular season; the sample pertinent to this application is 158 players. The players have been classified into five groups according to their position: playmaker, guard, small forward, power forward, and center, with the purpose of ensuring homogeneous samples when assessing the relative efficiency of the players.

As for the selection of variables, the application includes the same indicators as in Cooper et al. (2009), which reflect the main aspects of the game (shooting, rebounding, ball handling, and defense), as follows:

1. Adjusted field goal (AFG) = $(PTS - FTM) \times AFG\%$, where PTS = points made (per game), FTM = free throws made (per game), and AFG% (adjusted field goal percentage) = $\frac{\text{PTS} - \text{FTM}}{2 \times \text{FGA}}$, where FGA = number of field goal attempts. AFG% is used in NBA statistics for the purpose of measuring shooting efficiency by taking into account the total points a player produces through his field goal attempts. The intuition behind this adjustment is largely to evaluate the impact of 'three-point shooting'. Therefore, AFG is a shooting indicator adjusted for opportunities.
2. Adjusted free throw (AFT) = FTM × FT%, where FT% = free throw successes percentage.
3. Rebounds (REB) = number of rebounds per game.
4. Assists (AST) = number of assists per game.
5. Steals (STE) = number of steals per game.
6. Inverse of turnovers (ITURN) = This application involved using the inverse of the number of turnovers per game to treat the information regarding this indicator as an output that decreases with increases in turnovers, instead of an input. The approach is necessary to obtain an index of the same form as the one evident in the ACB League.
7. Nonmade fouls own (NFO) = 5 – FO, with FO = number of fouls made (per game) by the assessed player. The purpose of this transformation is the same as in ITURN.
8. Fouls opposite (FOPP) = number of fouls per game the opposite players have made on the assessed player.

The DEA model used is the output-oriented CCR model with a constant input and with outputs being the indicators above described[2] (note that in this case, the cross-efficiencies would be the inverse of those in Model 3, which are greater than or equal to 1). No additional distinctions regarding a specific playing style for the players in a given position (e.g., an assessment of their defensive or offensive games) are included but could be by means of a specific selection of variables, by using other model players purposely selected or by imposing some preferences over these variables with weight restrictions. In the latter respect, note that in contrast to Cooper et al. (2009), the approach within this paper does not include the opinion of experts on the relative importance of different aspects of the game. In the present paper, the expertise is useful only to identify some model players, whose weights are required to set a limit for the allowable differences in weights in the assessments of the players.

The purpose of the application is to make a cross-efficiency evaluation of the players in the position of center. The sample of centers is 35 players. Table 1 reflects the data together with the CCR efficiency scores. The ACB MVP awards (2008-2009) were useful in selecting the model players. Based on the opinion of coaches, players, media, and supporters, the ACB League awards MVP status to the player with the best score obtained as the sum of the voters' scores. The MVP results corresponding to the 2008-2009 season appear in Table 2.

Table 1
Data and CCR Efficiency Scores

Player	CCR score	AFG	AFT	REB	AST	STE	ITURN	NFO	FOPP
Bueno, A.	1.1744	5.76	1.26	4.13	0.66	1.00	0.52	2.28	3.03
Ramos, P.J.	1.3821	3.34	0.30	4.04	0.15	0.19	0.57	2.31	1.04
Triguero, J.	1.2202	2.27	1.60	4.25	0.56	0.94	0.86	2.00	2.44
Savanovic, D.	1.0000	3.75	1.39	3.57	1.20	0.83	0.67	3.20	2.37
Hdez-Sonseca, E.	1.1568	3.23	1.72	5.43	0.87	0.97	0.61	2.47	2.17
Wideman, T.	1.3055	3.85	1.99	4.19	0.72	0.78	0.50	2.09	3.13
Popovic, P.	1.1137	4.97	1.00	3.06	0.75	0.31	0.67	2.78	1.72
Junyent, O.	1.0000	1.57	1.01	2.67	0.96	0.33	1.20	3.08	1.54
Vázquez, F.	1.0000	6.56	2.09	6.65	0.68	0.97	1.29	3.03	2.61
Andersen, D.	1.1599	3.50	1.49	3.58	1.06	0.39	0.74	2.58	1.97
Santiago, D.	1.2297	2.37	0.94	1.84	0.63	0.31	0.64	2.56	2.03
Archibald, R.	1.2115	3.81	1.75	4.31	0.63	0.78	0.52	1.63	4.22
Ndong, B.	1.0865	3.90	2.05	4.88	0.81	0.84	0.76	2.72	2.56
Doblas, D.	1.6849	4.08	1.34	4.38	1.06	0.56	0.37	1.31	2.94
Marconato, D.	1.0417	1.82	0.39	3.42	0.48	0.35	1.24	2.84	0.77
Borchardt, C.	1.0000	5.67	1.68	9.88	1.08	0.65	0.39	2.15	5.46
Maric, A.	1.1054	1.66	0.96	3.50	0.38	0.38	0.94	2.88	1.84
Guardia, S.	1.1717	3.27	1.20	3.72	0.72	0.75	0.89	2.59	2.41
Perovic, K.	1.1197	4.26	1.59	3.94	0.47	0.84	0.67	2.72	2.75
Miralles, A.	1.1816	2.28	0.57	3.31	0.41	0.94	0.89	2.34	1.31
Kuqo, E.	1.3572	2.27	0.86	1.91	0.50	0.23	0.73	2.36	1.14
Asselin, J.	1.0875	6.31	1.46	4.84	1.00	0.94	0.49	2.13	2.48
Rubio, G.	1.1071	3.05	2.27	4.06	0.84	0.52	0.49	2.52	3.74
Ibaka, S.	1.0000	3.54	0.68	4.60	0.20	0.33	1.03	3.33	1.10
Alzamora, A.	1.0405	0.72	0.88	2.58	0.52	0.35	1.24	2.77	2.52
Eley, B.	1.1998	3.33	1.26	5.74	0.83	1.22	0.43	1.26	3.26
Fernández, J.	1.0163	1.35	0.40	2.20	0.17	0.47	1.20	3.07	1.10
Garcés, R.	1.2051	1.63	0.55	4.25	0.19	0.56	1.03	2.56	1.66
Barnes, L.	1.2289	3.73	1.04	6.25	0.78	0.97	0.64	2.25	1.72
Fajardo, D.	1.2156	3.19	1.16	5.91	1.19	1.00	0.58	2.25	1.88
Savané, S.	1.1490	4.36	1.26	4.66	1.19	0.72	0.71	2.50	2.34
Reyes, F.	1.0000	6.79	2.77	9.40	1.77	1.30	0.57	2.63	5.33
Splitter, T.	1.0000	7.00	2.63	6.27	2.15	1.50	0.58	1.73	5.19
McDonald, W.	1.2565	3.47	0.39	4.06	0.63	0.44	1.00	1.94	1.06
Barac, S.	1.0109	0.92	0.35	3.09	0.52	0.26	1.28	2.57	0.57

Table 2
MVP Award Scores (Season 2008-2009)

Player	Position	Total	Coaches	Players	Media	Supporters
Felipe Reyes	Center	90	25	25	25	15
Igor Rakocevic	Guard	75	25	10	15	25
Tiago Splitter	Center	40	15	15	10	0
Fran Vázquez	Center	40	10	15	5	10
Juan Carlos Navarro	Guard	23	5	10	3	5

Note. Retrieved from http://www.acb.com

Table 2 shows the five players selected for the final election of the MVP. Felipe Reyes won the 2008-2009 ACB MVP award with a total score of 90 points. Three of the five players were centers: Felipe Reyes, Tiago Splitter, and Fran Vázquez. These three players served as the model DMUs in the current analysis because they were the best players in that position in the opinion of the four communities involved in developing the ACB League.[3]

The Cross-Efficiency Evaluation

Let M:={Reyes, Splitter, Vázquez} be the set of model players. The optimal absolute weights provided by both Model 1 and Model 2 (Model 1 for the model players and Model 2, after setting $\varphi^* = 0.18388$, for the remaining players) appear in Table 3.

Table 3
Cross-Efficiency Evaluation: Absolute Weights

Player	AFG	AFT	REB	AST	STE	ITURN	NFO	FOPP	φ_d^*
Bueno, A.	0.03963	0.03322	0.01018	0.06398	0.22833	0.28033	0.10009	0.01385	
Ramos, P.J.	0.06317	0.12877	0.05229	0.25239	0.20191	0.37361	0.09151	0.03739	
Triguero, J.	0.02071	0.02933	0.01104	0.08340	0.27215	0.29501	0.12757	0.01925	
Savanovic, D.	0.01336	0.03610	0.01404	0.17099	0.06007	0.40838	0.08508	0.02115	
Hdez-Sonseca, E.	0.01576	0.02959	0.00936	0.05870	0.28621	0.31410	0.11216	0.02348	
Wideman, T.	0.01376	0.14455	0.01265	0.07368	0.06779	0.31836	0.13756	0.01695	
Popovic, P.	0.05015	0.04569	0.01495	0.09303	0.14653	0.37353	0.08954	0.02664	
Junyent, O.	0.03782	0.05846	0.02222	0.06183	0.17776	0.26853	0.10451	0.03843	
Vázquez, F.	0.00904	0.02838	0.00892	0.08747	0.06123	0.24948	0.10627	0.02268	0.18388
Andersen, D.	0.01546	0.03640	0.01512	0.13173	0.13989	0.39900	0.11412	0.02752	
Santiago, D.	0.02454	0.06166	0.03155	0.12235	0.18614	0.49429	0.12345	0.02864	
Archibald, R.	0.01631	0.03554	0.01442	0.09952	0.07962	0.55899	0.03828	0.08018	
Ndong, B.	0.01248	0.12944	0.00999	0.05991	0.05770	0.29808	0.09738	0.01900	
Doblas, D.	0.05299	0.02976	0.00909	0.15257	0.07072	0.58814	0.05302	0.07364	
Marconato, D.	0.03255	0.15307	0.01733	0.12246	0.16699	0.25987	0.11352	0.07654	
Borchardt, C.	0.01045	0.03535	0.03260	0.05502	0.09062	0.15041	0.02751	0.05900	
Maric, A.	0.03565	0.06180	0.01693	0.15801	0.15801	0.34238	0.11208	0.03214	
Guardia, S.	0.01759	0.04791	0.01545	0.12229	0.07660	0.35148	0.12045	0.02387	
Perovic, K.	0.01105	0.02964	0.01195	0.10042	0.30340	0.37900	0.09416	0.01712	
Miralles, A.	0.02057	0.08212	0.01416	0.11548	0.27215	0.28703	0.10886	0.03574	
Kuqo, E.	0.02606	0.06861	0.03104	0.11851	0.26071	0.43942	0.13633	0.05214	
Asselin, J.	0.03704	0.02938	0.00889	0.04300	0.24998	0.25699	0.10984	0.01731	
Rubio, G.	0.01293	0.09457	0.00970	0.04703	0.07642	0.40385	0.08525	0.05732	
Ibaka, S.	0.06996	0.06667	0.01607	0.22793	0.13676	0.23965	0.07437	0.04144	
Alzamora, A.	0.08200	0.06709	0.02296	0.11480	0.16699	0.25987	0.11616	0.02355	
Eley, B.	0.02114	0.05581	0.01406	0.08514	0.31418	0.43474	0.05578	0.02157	
Fernández, J.	0.04383	0.14762	0.02693	0.35552	0.12697	0.26853	0.10508	0.05387	
Garcés, R.	0.03631	0.10852	0.01394	0.31602	0.10534	0.31217	0.12575	0.03578	
Barnes, L.	0.01122	0.04020	0.02384	0.05362	0.23515	0.35595	0.10125	0.02437	
Fajardo, D.	0.01308	0.03584	0.00706	0.12866	0.22680	0.38981	0.10080	0.02224	
Savané, S.	0.01158	0.03994	0.01084	0.16717	0.07024	0.38607	0.10982	0.02154	
Reyes, F.	0.01840	0.04509	0.01330	0.07075	0.09615	0.22083	0.04747	0.02344	1.00000
Splitter, T.	0.02765	0.02399	0.01008	0.08979	0.12894	0.28923	0.03650	0.01217	0.32662
McDonald, W.	0.01706	0.15169	0.01459	0.09480	0.13544	0.32224	0.16632	0.05577	
Barac, S.	0.06450	0.16894	0.01919	0.11357	0.22714	0.25219	0.12562	0.10483	

These weight profiles have the least dissimilar virtuals in the sense that the lowest input (output) virtual is not lower than 18.39% of the highest one. Thus, the approach avoids the profiles with the largest differences in the roles that the different aspects of the game play in the assessment of the effectiveness of players in an attempt to prevent unrealistic weighting schemes. This, in particular, has guaranteed nonzero weights, as evident in the table, which means that no aspect of the game has been ignored in the assessment.

The values φ_d^* for the model players in Table 3 show how much they need to unbalance the importance attached to the different aspects of the game to be rated as effective players. The value $\varphi_d^* = 1$ for Felipe Reyes, the MVP, indicates that he could be assessed as an effective player even with a set of weights in which all the variables would contribute equally to the score of effectiveness. Thus, Reyes is a good player in all aspects of the game. Tiago Splitter, with $\varphi_d^* = 0.32662$, needs to unbalance his virtual weights: With a profile of weights in which the minimum virtual was greater than 32.66% of the maximum virtual, he would not be classified as an effective player. Fran Vázquez is the model player who needs to use the most dissimilar virtuals to be rated as effective. Therefore, his value, $\varphi_d^* = 0.18388$, is actually that used to set φ^*, which is the limit for the allowable differences in virtual weights used in the assessment of all the players. Therefore, all the players are permitted to choose their weights freely provided that the minimum virtual is not lower than 18.39% of the maximum virtual.

Note that the CCR model revealed four other players as relative effective centers: Savanovic, Junyent, Borchardt, and Ibaka. Obviously, basketball experts would not agree with this result. From a technical point of view, these players took advantage of DEA total weight flexibility to be rated as effective players. Solving Model 1 for them provides their φ_d^*'s (because these are extreme efficient points on the frontier) and alternate optima for their weights, with at least one of these optimal solutions having nonzero weights. The values are 0.0269, 0.0136, 0.0231, and 0.0079, respectively, which show that the players need to unbalance their virtual weights drastically to be assessed as effective. Using these players in the proposed approach to specify the value φ^*, which would be $\varphi^* = 0.0079$, would allow for larger differences in the virtual weights (still guaranteeing nonzero weights). Note that as φ^* tends to 0, the results obtained move closer to the results that the CCR model would provide.

Table 4 illustrates the matrix of cross-efficiencies calculated with the optimal weights from Table 3 and the corresponding cross-efficiency scores, which are the averages of the cross-efficiencies in each column (top 15 players reported to conserve space). The corresponding ranking of centers appears in Table 5.

Cross-efficiency evaluation facilitates an ordering of the model players. With this approach, the cross-efficiency score of the MVP, Felipe Reyes, equals 1, which means that all the players have evaluated him as an effective player. The other two model players, Splitter and Vázquez, also exhibit a high cross-efficiency score, 1.045696 and 1.0577046, respectively, which shows that the rest of the players provided good evaluations of them as well.

As for the remaining players, Borchardt ranks fourth, with a substantially lower cross-efficiency score of 1.321099. Basketball experts would not be surprised that Borchardt ranks high seeing that he has caught more rebounds and provoked more fouls than any other center. He further won the MVP award for several weeks and months. Ndong, who ranks fifth, also played a good season. After him, Savanovic, initially rated as effective, ranks sixth. The other players assessed as effective under the CCR model now rank far lower under the cross-efficiency evaluation because the approach stops them from taking advantage of DEA total weight flexibility.[4] In general, all the results are consistent with ACB basketball expert opinion.

Table 4
Matrix of Cross-Efficiencies and Cross-Efficiency Scores (Top 15 Players)

Player	Reyes	Splitter	Vázquez	Borchardt	Ndong	Savanov	Savané	Asselin	Guardia	H-Sons	Junyent	Perovic	Bueno	Fajardo	Triguero
Bueno, A.	1.0000	1.0357	1.0000	1.3617	1.3131	1.3132	1.3849	1.3481	1.4103	1.4123	1.4708	1.3889	1.3630	1.4443	1.4935
Ramos, P.J.	1.0000	1.0453	1.1699	1.2910	1.4909	1.5487	1.5441	1.5047	1.7048	1.5903	1.7992	1.6815	1.6203	1.5939	1.7587
Triguero, J.	1.0000	1.0453	1.0171	1.3779	1.2872	1.2621	1.3679	1.3912	1.3677	1.3648	1.3783	1.3729	1.3868	1.3890	1.4325
Savanovic, D.	1.0000	1.0350	1.0000	1.3266	1.2961	1.2710	1.3128	1.4661	1.3497	1.4226	1.1947	1.4743	1.5402	1.4133	1.4524
Hdez-Sonseca, E.	1.0000	1.0393	1.0000	1.3853	1.2733	1.2672	1.3734	1.3985	1.3442	1.3513	1.3646	1.3528	1.3741	1.3859	1.3825
Wideman, T.	1.0000	1.1038	1.0000	1.3517	1.2325	1.2906	1.3975	1.4815	1.3950	1.3788	1.2989	1.3781	1.5213	1.5118	1.4456
Popovic, P.	1.0000	1.0387	1.0000	1.3039	1.3409	1.3651	1.3937	1.3777	1.4380	1.4764	1.4531	1.4398	1.4149	1.5111	1.5412
Junyent, O.	1.0000	1.0718	1.0699	1.2744	1.3763	1.4178	1.4741	1.4417	1.5152	1.4898	1.6004	1.4693	1.4581	1.5471	1.5845
Vázquez, F.	1.0000	1.0871	1.0000	1.2950	1.2554	1.2180	1.3143	1.4455	1.3202	1.3786	1.2006	1.3719	1.4590	1.4157	1.4410
Andersen, D.	1.0000	1.0532	1.0000	1.3236	1.2839	1.2669	1.3365	1.4440	1.3459	1.3947	1.2514	1.4147	1.4698	1.4107	1.4313
Santiago, D.	1.0000	1.0740	1.0000	1.2983	1.3049	1.3364	1.3825	1.4550	1.3937	1.4103	1.3383	1.4397	1.4876	1.4377	1.4500
Archibald, R.	1.0000	1.0220	1.0000	1.2299	1.3477	1.4228	1.4130	1.5391	1.3732	1.5212	1.2951	1.4783	1.5193	1.5661	1.4110
Ndong, B.	1.0000	1.0865	1.0000	1.3536	1.2448	1.3283	1.4194	1.4968	1.4103	1.4009	1.3226	1.3979	1.5372	1.5438	1.4401
Doblas, D.	1.0000	1.0000	1.0000	1.2543	1.3747	1.4034	1.3858	1.4313	1.4120	1.5630	1.3603	1.4887	1.4575	1.5827	1.5224
Marconato, D.	1.0000	1.0492	1.1824	1.2951	1.4282	1.4974	1.5767	1.5506	1.6412	1.5702	1.7392	1.5635	1.5794	1.6812	1.6688
Borchardt, C.	1.0000	1.0908	1.2696	1.1627	1.5846	1.7252	1.6937	1.6745	1.7825	1.6737	2.0104	1.7391	1.6934	1.7024	1.7521
Maric, A.	1.0000	1.0383	1.0726	1.3228	1.3657	1.3589	1.4166	1.4431	1.4833	1.4823	1.4634	1.4996	1.4965	1.5029	1.5798
Guardia, S.	1.0000	1.0768	1.0000	1.3009	1.2770	1.2628	1.3347	1.4454	1.3567	1.4027	1.2446	1.4114	1.4843	1.4354	1.4653
Perovic, K.	1.0000	1.0165	1.0000	1.4199	1.2821	1.2765	1.3630	1.4169	1.3440	1.3503	1.3312	1.3952	1.4161	1.3580	1.3652
Miralles, A.	1.0000	1.0318	1.1053	1.3701	1.3618	1.3793	1.4739	1.4761	1.5042	1.4521	1.5582	1.4855	1.4949	1.4988	1.5280
Kuqo, E.	1.0000	1.0655	1.0624	1.2927	1.3516	1.3811	1.4451	1.4808	1.4605	1.4498	1.4723	1.4692	1.4919	1.4868	1.5041
Asselin, J.	1.0000	1.0437	1.0000	1.3591	1.3022	1.2992	1.3882	1.3446	1.3968	1.3962	1.4779	1.3609	1.3402	1.4382	1.4754
Rubio, G.	1.0000	1.0678	1.0000	1.2751	1.2798	1.3661	1.4304	1.5223	1.3813	1.4501	1.3187	1.3984	1.5004	1.5798	1.4115
Ibaka, S.	1.0000	1.0000	1.1892	1.3107	1.5199	1.5010	1.5113	1.4379	1.7051	1.6511	1.8258	1.6684	1.5418	1.6373	1.8570
Alzamora, A.	1.0000	1.0493	1.0760	1.2890	1.4212	1.4394	1.4654	1.3672	1.5910	1.5505	1.7327	1.5118	1.4319	1.5848	1.7379
Eley, B.	1.0000	1.0000	1.0000	1.4143	1.3163	1.3736	1.4204	1.4281	1.4019	1.4013	1.4392	1.4400	1.4383	1.4267	1.3871
Fernández, J.	1.0000	1.0132	1.2698	1.3316	1.5163	1.5055	1.5469	1.5559	1.7489	1.6323	1.7608	1.7467	1.6979	1.6317	1.8392
Garcés, R.	1.0000	1.0112	1.1833	1.3598	1.4307	1.3831	1.4478	1.5015	1.5986	1.5535	1.5244	1.6354	1.6246	1.5506	1.7266
Barnes, L.	1.0000	1.0770	1.0000	1.3185	1.2902	1.3278	1.4008	1.4541	1.3745	1.3704	1.3712	1.3982	1.4454	1.4064	1.3944
Fajardo, D.	1.0000	1.0138	1.0000	1.4053	1.2759	1.2532	1.3382	1.4217	1.3335	1.3699	1.2704	1.3994	1.4347	1.3815	1.3885
Savané, S.	1.0000	1.0450	1.0000	1.3380	1.2717	1.2312	1.3006	1.4544	1.3289	1.3960	1.1757	1.4320	1.5119	1.4011	1.4417
Reyes, F.	1.0000	1.0429	1.0700	1.3026	1.3797	1.4316	1.4644	1.4812	1.5097	1.5004	1.5213	1.5213	1.5253	1.5407	1.5544
Splitter, T.	1.0000	1.0000	1.0000	1.3575	1.3540	1.3766	1.3842	1.3907	1.4310	1.4681	1.4247	1.4883	1.4474	1.4613	1.4970
McDonald, W.	1.0000	1.0863	1.0827	1.3113	1.3076	1.3512	1.4663	1.5200	1.4730	1.4477	1.4431	1.4332	1.5291	1.5715	1.5225
Barac, S.	1.0000	1.0424	1.1995	1.2746	1.4720	1.5493	1.6134	1.5160	1.6997	1.6222	1.9238	1.5743	1.5362	1.7397	1.7441
$\bar{E}_j$	1.0000	1.0457	1.0577	1.3211	1.3489	1.3709	1.4281	1.4601	1.4665	1.4670	1.4673	1.4777	1.4935	1.5077	1.5322

Table 5
Ranking

Rank	Name	Cross-efficiency score	Rank	Name	Cross-efficiency score
1	Reyes, F.	1.00000	19	Alzamora, A.	1.56233
2	Splitter, T.	1.04570	20	Ibaka, S.	1.56657
3	Vázquez, F.	1.05770	21	Wideman, T.	1.57541
4	Borchardt, C.	1.32110	22	Archibald, R.	1.62114
5	Ndong, B.	1.34886	23	Marconato, D.	1.64152
6	Savanovic, D.	1.37091	24	Popovic, P.	1.66117
7	Savané, S.	1.42805	25	Eley, B.	1.67099
8	Asselin, J.	1.46013	26	Maric, A.	1.69189
9	Guardia, S.	1.46647	27	Fernández, J.	1.70276
10	Hdez-Sonseca, E.	1.46702	28	Miralles, A.	1.70369
11	Junyent, O.	1.46735	29	Garcés, R.	1.72055
12	Perovic, K.	1.47773	30	McDonald, W.	1.76130
13	Bueno, A.	1.49345	31	Barac, S.	1.76708
14	Fajardo, D.	1.50771	32	Doblas, D.	1.87050
15	Triguero, J.	1.53220	33	Santiago, D.	1.92531
16	Rubio, G.	1.53933	34	Kuqo, E.	2.06676
17	Barnes, L.	1.54537	35	Ramos, P.J.	2.31694
18	Andersen, D.	1.55841			

Analysis of Sensitivity

Analysis of the sensitivity of the results to the specification of the value φ^*, which represents the limit for the allowable differences in virtual weights in the assessment of the players, provided further insight. Because φ^* is initially set at 0.1838 (according to the differences in weights observed in the model players), repetition of the cross-efficiency evaluation for the values of φ^* around 0.1838, going from 0.1 to 0.25 by 0.025, was carried out. Table 6 indicates the rankings and the cross-efficiency scores of the players for each value of φ^*.

Reyes ranks first regardless of the specification of φ^*. The top eight players remain unchanged for values of φ^* greater than 0.1838 (i.e., more demanding with the differences in virtual weights allowed). Players, such as Guardia and Junyent (more specialized), reflect lower rankings as φ^* increases, while other players, such as Hernández-Sonseca, Bueno, and Perovic, maintain their positions or rank higher. When φ^* decreases (i.e., when more differences in virtual weights are allowed), Splitter and Vázquez exchange positions so that Vázquez ranks second and Splitter third. Borchardt and Ndong stand in fourth and fifth positions (except for the case $\varphi^* = 0.1$, where Savanovic ranks higher). As expected, some players, such as Junyent, rank higher under conventional DEA.

Comparison to Other Approaches

Using real data highlights some issues of interest hardly addressed in the literature regarding the choice of weights in cross-efficiency evaluations. The selection of weights for inefficient DMUs merits detailed analysis. In practice, many of the inefficient DMUs generally have a unique optimal solution for their weights in the DEA model, and more important, the solution has several zeros. Due to this uniqueness, in cross-efficiency evaluations based on a choice of weights among the alternate optima in the CCR model, these DMUs can only choose this optimal solution as weight profiles for calculation of the cross-efficiencies. Such is the case in many of the existing approaches in the literature. For example, in the benevolent and in the aggressive formulations, the secondary goal used is based on such a choice both in the cases of the efficient and inefficient DMUs, which try to globally maximize or minimize (respectively) the resulting cross-efficiencies.

Table 6
Analysis of Sensitivity (Rankings)

$\varphi^* = 0.1$		$\varphi^* = 0.125$		$\varphi^* = 0.15$		$\varphi^* = 0.1838$		$\varphi^* = 0.2$		$\varphi^* = 0.225$		$\varphi^* = 0.25$	
Player	Score	Player	Score	Player	Score	Player	Score	Player	Score	Player	Score	Player	Score
Reyes, F.	1,00	Reyes, F.	1,00	Reyes, F.	1,00	Reyes, F.	1,00	Reyes, F.	1,00	Reyes, F.	1,00	Reyes, F.	1,00
Vázquez, F.	1,02	Vázquez, F.	1,03	Vázquez, F.	1,05	Splitter, T.	1,05	Splitter, T.	1,04	Splitter, T.	1,04	Splitter, T.	1,04
Splitter, T.	1,06	Splitter, T.	1,06	Splitter, T.	1,05	Vázquez, F.	1,06	Vázquez, F.	1,06	Vázquez, F.	1,08	Vázquez, F.	1,09
Savanov., D.	1,30	Ndong, B.	1,32	Borchardt, C.	1,32	Borchardt, C.	1,32	Borchardt, C.	1,32	Borchardt, C.	1,31	Borchardt, C.	1,31
Ndong, B.	1,30	Borchardt, C.	1,32	Ndong, B.	1,33	Ndong, B.	1,35	Ndong, B.	1,36	Ndong, B.	1,37	Ndong, B.	1,39
Borchardt, C.	1,33	Savanov., D.	1,33	Savanov., D.	1,35	Savanov., D.	1,37	Savanov., D.	1,38	Savanov., D.	1,41	Savanov., D.	1,42
Junyent, O.	1,35	Junyent, O.	1,39	Savané, S.	1,41	Savané, S.	1,43	Savané, S.	1,44	Savané, S.	1,46	Savané, S.	1,46
Savané, S.	1,37	Savané, S.	1,39	Junyent, O.	1,42	Asselin, J.	1,46	Asselin, J.	1,46	Asselin, J.	1,47	Asselin, J.	1,47
Guardia, S.	1,40	Guardia, S.	1,42	Guardia, S.	1,44	Guardia, S.	1,47	H-Sons., E.	1,48	H-Sons., E.	1,49	H-Sons., E.	1,50
H-Sons., E.	1,42	H-Sons., E.	1,44	H-Sons., E.	1,45	H-Sons., E.	1,47	Guardia, S.	1,48	Perovic, K.	1,51	Bueno, A.	1,51
Perovic, K.	1,43	Perovic, K.	1,44	Asselin, J.	1,46	Junyent, O.	1,47	Perovic, K.	1,49	Guardia, S.	1,51	Perovic, K.	1,52
Alzamora, A.	1,44	Asselin, J.	1,45	Perovic, K.	1,46	Perovic, K.	1,48	Junyent, O.	1,49	Bueno, A.	1,51	Guardia, S.	1,52
Ibaka, S.	1,44	Alzamora, A.	1,47	Bueno, A.	1,49	Bueno, A.	1,49	Bueno, A.	1,50	Fajardo, D.	1,53	Fajardo, D.	1,55
Asselin, J.	1,44	Ibaka, S.	1,48	Fajardo, D.	1,49	Fajardo, D.	1,51	Fajardo, D.	1,52	Junyent, O.	1,54	Rubio, G.	1,57
Fajardo, D.	1,46	Fajardo, D.	1,48	Alzamora, A.	1,51	Triguero, J.	1,53	Rubio, G.	1,55	Rubio, G.	1,56	Junyent, O.	1,57
Bueno, A.	1,47	Bueno, A.	1,48	Triguero, J.	1,51	Rubio, G.	1,54	Triguero, J.	1,55	Triguero, J.	1,57	Triguero, J.	1,58
Andersen, D.	1,48	Triguero, J.	1,50	Ibaka, S.	1,52	Barnes, L.	1,55	Barnes, L.	1,56	Barnes, L.	1,57	Barnes, L.	1,58
Triguero, J.	1,49	Andersen, D.	1,51	Rubio, G.	1,52	Andersen, D.	1,56	Andersen, D.	1,57	Wideman, T.	1,59	Wideman, T.	1,60
Rubio, G.	1,49	Rubio, G.	1,51	Andersen, D.	1,53	Alzamora, A.	1,56	Wideman, T.	1,58	Andersen, D.	1,60	Andersen, D.	1,61
Marcon., D.	1,49	Barnes, L.	1,52	Barnes, L.	1,53	Ibaka, S.	1,57	Ibaka, S.	1,59	Archib., R.	1,63	Archib., R.	1,62
Barnes, L.	1,50	Marcon., D.	1,54	Wideman, T.	1,57	Wideman, T.	1,58	Alzamora, A.	1,60	Ibaka, S.	1,63	Ibaka, S.	1,66
Fernández, J.	1,53	Wideman, T.	1,56	Marcon., D.	1,58	Archib., R.	1,62	Archib., R.	1,62	Alzamora, A.	1,64	Eley, B.	1,67
Wideman, T.	1,55	Fernández, J.	1,58	Archib., R.	1,62	Marcon., D.	1,64	Popovic, P.	1,67	Eley, B.	1,67	Alzamora, A.	1,67
Maric, A.	1,56	Maric, A.	1,60	Popovic, P.	1,63	Popovic, P.	1,66	Eley, B.	1,67	Popovic, P.	1,70	Popovic, P.	1,72
Popovic, P.	1,57	Popovic, P.	1,60	Fernández, J.	1,63	Eley, B.	1,67	Marcon., D.	1,68	Marcon., D.	1,73	Marcon., D.	1,77
Garcés, R.	1,59	Archib., R.	1,62	Maric, A.	1,64	Maric, A.	1,69	Maric, A.	1,72	Maric, A.	1,76	Miralles, A.	1,79
Barac, S.	1,60	Garcés, R.	1,63	Miralles, A.	1,66	Fernández, J.	1,70	Miralles, A.	1,73	Miralles, A.	1,77	Maric, A.	1,79
Miralles, A.	1,61	Miralles, A.	1,64	Garcés, R.	1,67	Miralles, A.	1,70	Fernández, J.	1,75	Garcés, R.	1,79	Garcés, R.	1,82
Archib., R.	1,62	Barac, S.	1,64	Eley, B.	1,68	Garcés, R.	1,72	Garcés, R.	1,75	Fernández, J.	1,81	McDon., W.	1,84
McDon., W.	1,66	Eley, B.	1,69	Barac, S.	1,69	McDon., W.	1,76	McDon., W.	1,78	McDon., W.	1,82	Fernández, J.	1,85
Eley, B.	1,70	McDon., W.	1,69	McDon., W.	1,72	Barac, S.	1,77	Barac, S.	1,81	Doblas, D.	1,87	Doblas, D.	1,87
Santiago, D.	1,79	Santiago, D.	1,84	Doblas, D.	1,87	Doblas, D.	1,87	Doblas, D.	1,87	Barac, S.	1,88	Barac, S.	1,92
Doblas, D.	1,86	Doblas, D.	1,87	Santiago, D.	1,87	Santiago, D.	1,93	Santiago, D.	1,95	Santiago, D.	1,99	Santiago, D.	2,02
Kuqo, E.	1,91	Kuqo, E.	1,96	Kuqo, E.	2,01	Kuqo, E.	2,07	Kuqo, E.	2,10	Kuqo, E.	2,15	Kuqo, E.	2,19
Ramos, P.J.	2,15	Ramos, P.J.	2,20	Ramos, P.J.	2,26	Ramos, P.J.	2,32	Ramos, P.J.	2,35	Ramos, P.J.	2,38	Ramos, P.J.	2,42

Table 7 includes the optimal weights provided by the CCR model for the ineffective players in the present application. Note that these are the unique solutions in the corresponding formulations. Many zeros are evident, which means that the cross-efficiencies calculated with the weight profiles provided by all these players ignore many of the variables initially considered for the analysis. In particular, the table shows that most of the ineffective players attach a zero weight to the variable REB, which means that their profiles ignore rebounding, which is a very important aspect of the game for a center. The same phenomenon is apparent with the weights of the variables concerning shooting, AFG and AFT, for many of the ineffective players, so their profiles will also disregard this crucial aspect of the game in the cross-efficiency calculation. In such situations, one cannot expect that the effects of these unrealistic weighting schemes are cancelled out in the summary of the cross-efficiency evaluation (the current analysis shows 28 inefficient players out of 35). Assessing the inefficient units using Model 2 allows for assessment of the effectiveness of these players with reference to Pareto-efficient points on the frontier and prevention of the optimal weights provided by the CCR model for them.

Table 7
Optimal DEA Weights for Ineffective Players

Player	AFG	AFT	REB	AST	STE	ITURN	NFO	FOPP
Bueno, A.	0.1406	0	0	0	0	0	0.0833	0
Ramos, P.J.	0	0	0.0495	0	0	0	0.3467	0
Triguero, J.	0	0	0	0	0.6293	0.4741	0	0
Hdez-Sonseca, E.	0	0	0	0	0.3310	0	0.2757	0
Wideman, T.	0	0.1807	0	0	0	0	0.3057	0
Popovic, P.	0.0242	0	0	0.0400	0	0	0.3056	0
Andersen, D.	0	0.0403	0	0.1691	0	0.1626	0.2480	0
Santiago, D.	0	0	0	0	0	0.0625	0.3239	0.0640
Archibald, R.	0	0	0	0	0	0.6006	0	0.1636
Ndong, B.	0	0.1467	0	0	0	0	0.2575	0
Doblas, D.	0.2254	0	0	0	0	0.0544	0.0533	0
Marconato, D.	0	0	0	0	0	0.8065	0	0
Maric, A.	0	0	0	0	0	0.0638	0.2957	0.0488
Guardia, S.	0	0	0	0	0	0.0563	0.3084	0.0623
Perovic, K.	0	0	0	0	0	0.0600	0.2943	0.0582
Miralles, A.	0	0	0	0	0.5653	0.1913	0.1280	0
Kuqo, E.	0	0.0232	0	0.08	0	0.1364	0.3554	0
Asselin, J.	0.1473	0	0	0	0	0	0.0329	0
Rubio, G.	0	0.1499	0	0	0	0	0.2623	0
Alzamora, A.	0	0	0	0	0	0.5242	0	0.1391
Eley, B.	0	0	0.0401	0	0.6325	0	0	0
Fernández, J.	0	0	0	0	0	0.2583	0.2250	0
Garcés, R.	0	0	0	0	0	0.3100	0.2654	0
Barnes, L.	0	0	0	0	0.3510	0	0.2933	0
Fajardo, D.	0	0	0	0	0.3500	0	0.2889	0
Savané, S.	0.0115	0	0	0.2105	0	0.1828	0.2320	0
McDonald, W.	0	0	0	0.2720	0	0.8300	0	0
Barac, S.	0	0	0	0	0	0.7826	0	0

Table 8 includes the choice of weights with the profiles provided by the effective players using the well-known benevolent formulation to observe the effects of these weight profiles on a cross-efficiency evaluation.[5] Many zero weights are apparent in the profiles of the effective players as well (the aggressive formulation usually yields more zeros). Even Felipe Reyes has zeros in five variables and exhibits the highest virtual in NFO, 0.694. Obviously, such a profile of weights provides a misleading picture of Reyes. Note also that all the effective players attach a zero weight to the variables AFG and AFT, which means that the profiles provided by these players will ignore the aspects of shooting in the calculation of the corresponding cross-efficiencies.

Table 8
Weight Profiles for Effective Players (Benevolent Formulation)

Player	AFG	AFT	REB	AST	STE	ITURN	NFO	FOPP
Savanovic, D.	0	0	0	0	0	0.0502	0.2635	0.0520
Junyent, O.	0	0	0	0.1073	0	0.1391	0.2234	0.0269
Vázquez, F.	0	0	0	0	0	0.0502	0.2635	0.0520
Borchardt, C.	0	0	0.0910	0	0	0.2561	0	0
Ibaka, S.	0	0	0.0057	0	0	0.0279	0.2698	0.0412
Reyes, F.	0	0	0	0	0	0.0502	0.2635	0.0520
Splitter, T.	0	0	0	0.2352	0.0678	0.3181	0.1201	0

Table 9 indicates the cross-efficiency scores and the associated rankings corresponding to the weight profiles of the benevolent formulation. To be precise, the cross-efficiency scores are the average of the cross-efficiencies, calculated using the weights in Tables 7 and 8 depending on whether the corresponding player is effective or ineffective. As expected, this approach provides results that are not consistent with the ACB expert views: Fran Vázquez ranks first, with a cross-efficiency score of 1.012572, which is substantially better than that of Reyes, at 1.113199, who now ranks second. In addition, some players, such as Savanovic and Ibaka, are at the top of the ranking before other players, such as Splitter.

Table 9
Ranking (Benevolent Formulation)

Rank	Player	Cross-efficiency score	Rank	Player	Cross-efficiency score
1	Vázquez, F.	1.0126	19	Barnes, L.	1.4758
2	Reyes, F.	1.1132	20	Garcés, R.	1.4874
3	Savanovic, D.	1.2324	21	Fajardo, D.	1.4910
4	Ibaka, S.	1.2519	22	Asselin, J.	1.4943
5	Ndong, B.	1.2705	23	Rubio, G.	1.4993
6	Splitter, T.	1.3113	24	Miralles, A.	1.5051
7	Perovic, K.	1.3276	25	Triguero, J.	1.5357
8	Guardia, S.	1.3308	26	Alzamora, A.	1.5435
9	Savané, S.	1.3539	27	Wideman, T.	1.5668
10	Junyent, O.	1.3602	28	McDonald, W.	1.6435
11	Fernández, J.	1.4098	29	Barac, S.	1.6461
12	Hdez-Sonseca, E.	1.4110	30	Santiago, D.	1.6665
13	Marconato, D.	1.4137	31	Archibald, R.	1.7104
14	Bueno, A.	1.4434	32	Kuqo, E.	1.7830
15	Andersen, D.	1.4486	33	Ramos, P.J.	1.8489
16	Borchardt, C.	1.4522	34	Eley, B.	1.9535
17	Maric, A.	1.4548	35	Doblas, D.	2.0896
18	Popovic, P.	1.4624			

Conclusions

The purpose of this paper was to extend the multiplier bound approach (Ramón et al., 2010a) for use in cross-efficiency evaluations with the aim of preventing unrealistic weighting schemes. The proposed approach involved guaranteeing nonzero weights and attempting to avoid profiles with large differences in weights. In the case developed the limit for the allowable differences in weights is specified from those observed in some model DMUs.

The paper included an application of the proposed approach to the ranking of basketball players in the context of the Spanish basketball league (the ACB League); available information allowed for the identification of some model players. Care is required in the choice of the weights made by the inefficient DMUs in cross-efficiency evaluations. In practice, many of these DMUs have a unique optimal solution with several zeros for their weights in the CCR model; therefore, these weight profiles will be those used in the cross-efficiency evaluations based on a choice among the alternate optima provided by this model. As evident in the application, the weights may have an undesired effect on the resulting cross-efficiency scores and lead to conclusions not consistent with prior knowledge or expert opinion. Thus, in the approach proposed, the inefficient DMUs are reassessed with a model that prevents from using the weights provided by the CCR model. The approach produced satisfactory results in the case of ranking basketball players.

Footnotes

1 According to the classification of units in Charnes, Cooper, and Thrall (1991), the DMUs in E and E´ are Pareto-efficient. E consists of the extreme efficient units, whereas E´ relates to Pareto-efficient units that can be expressed as a combination of DMUs in E. F is the set of weakly efficient units. The DMUs in NE, NE´, and NF are inefficient and are projected onto points on the frontier that are in E, E´, and F, respectively.

2 A departure is apparent here from the more customary efficiency analyses to a focus on effectiveness in the sense that the analysis includes no reference to resources consumed (see Prieto & Zofio, 2001) as in the efficiency evaluations evident in microeconomics. The approach confines attention to player outputs, such as points scored and/or percentage of free-throw successes, and leaves out elements such as player salaries, but does not include benefit measures, such as revenues earned, or similar considerations.

3 Note that the approach followed in this paper would not work with, for example, the playmakers because information on model players in that position is not available.

4 Note that the four nonmodel players initially rated as effective now become ineffective even with their own weights in Model 2. For example, see the case of Borchardt (1.1627), Savanovic (1.2710), and Junyent (1.6004) in Table 4.

5 These weights have been obtained by means of a formulation (Liang et al., 2008a) that is equivalent to using as surrogate the differences between the numerator and the denominator of the CCR efficiency score in ratio form.

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Authors Note

William W. Cooper, The Red McCombs School of Business, The University of Texas at Austin,1 University Station B6500, Austin, TX 78712-0212, USA.

Nuria Ramón, José L. Ruiz, and Inmaculada Sirvent, Centro de Investigación Operativa, Universidad Miguel Hernández, Avd. de la Universidad, s/n, 03202-Elche (Alicante), Spain.

Correspondence concerning this article should be addressed to José L. Ruiz, Email: jlruiz@umh.es

The authors are very grateful to Ministerio de Ciencia e Innovación (MTM2009-10479) and to Generalitat Valenciana (ACOMP/2011/115) for the financial support.